Answer all questions from section B. Questions in section B carry $16\frac{2}{3}$ marks each. Students are allowed to use course notes, books and calculators.
Section B: Functional Programming

4. This question deals with lists:

(a) Write a function \texttt{choose}, which given a list of integers \texttt{ps} and another list \texttt{x}s, returns the elements of \texttt{x}s in the positions given in \texttt{ps}. For example, \texttt{choose [1,5,2] "Hello world"} will return the first, fifth and second elements of "Hello world", namely "Hol".

(b) A self-important list of integers is one which when used to choose elements from itself (see question 1), will return the original list once again. Write a function which checks whether a given list of integers is self-important.

(c) Given a function \texttt{f} of type \texttt{Int -> a} (for some type \texttt{a}), we can induce a list of \texttt{a}, by applying \texttt{f} to 1, 2, 3, etc. For example, the list induced by the \texttt{factorial} function is [1,2,6,24,...]. Write a function which given a function of the correct type, returns the induced list.

5. Give the full type of the following functions:

\begin{verbatim}
one f xs = map (f xs) xs
three f x = f x
two f x = if (x == f x) then x else f x
\end{verbatim}

6. Consider the following datatype:

\texttt{data Direction = N | S | E | W}

(a) Without using \texttt{deriving}, make \texttt{Direction} an instance of the typeclass \texttt{Eq}.

(b) Define a function \texttt{opposite}, which given a direction, returns the opposite direction.

(c) A list of directions is said to be in normal form if it contains no opposites. Write a function which checks whether a given list of integers is in normal form.