CSA2140: Formal Languages and Automata  31 January 2004

09:15–11:45

Answer any three questions. Each question carries 33\(\frac{1}{2}\) marks. Students are allowed to use course notes, books and calculators.

1. (a) Using standard constructions, give a deterministic finite state automaton equivalent to the following one:

![Automaton](image)

(b) Using standard constructions, obtain a regular expression and a regular grammar equivalent to the following automaton:

![Automaton](image)

2. Let grammar \(G\) over alphabet \{0, 1\} and with start symbol \(S\) have the following production rules:

\[ S \rightarrow 00S1 \mid \varepsilon \]

(a) Using standard constructions, give a pushdown automaton which accepts the same language as \(G\).

(b) Prove that \( \mathcal{L}(G) \subseteq \{0^{2n}1^n \mid n \in \mathbb{N}\} \cup \{0^{2n}S1^n \mid n \in \mathbb{N}\} \).

(c) Hence prove that \( \mathcal{L}(G) \subseteq \{0^{2n}1^n \mid n \in \mathbb{N}\} \).
3. (a) Consider the two grammars (both with start symbol $S$):

Grammar 1 ($G_1$):
- $S \rightarrow 1S1 \mid + A$
- $A \rightarrow 1A1 \mid =$

Grammar 2 ($G_2$):
- $S \rightarrow 1S1 \mid A+$
- $A \rightarrow 1A1 \mid =$

Using standard constructions, give a context-free grammar which accepts the language $L(G_1) \cup L(G_2)$.

(b) Prove that $\forall n \in \mathbb{N} \cdot (ab)^na = a(ba)^n$.

(c) Using the result in (b), prove that the regular expressions $a(ba)^*$ and $(ab)^*a$ are equivalent.

4. (a) Draw the pushdown automaton described below. Then, using standard constructions, convert it into one with one final state, and satisfying the property that the stack is emptied upon (and only upon) termination.

$$
\Sigma = \{a, b\} \\
\Gamma = \{A\} \\
Q = \{\alpha, \beta\} \\
q_0 = \alpha \\
F = \{\alpha, \beta\} \\
t = \{(\alpha, (a, \lambda, A), \alpha), \\
(\alpha, (b, A, \varepsilon), \beta), \\
(\beta, (b, A, \varepsilon), \beta), \\
(\beta, (a, \lambda, A), \alpha)\}
$$

Recall that a transition $(x, (a, b, s), y)$ goes from state $x$ to state $y$ reading symbol $a$ from the input, $b$ from the stack, and writing string $s$ back onto the stack. If $b$ is $\lambda$, this indicates that nothing is read off the stack.

(b) Let grammar $G$ over alphabet $\{a, b\}$ and with start symbol $S$ have the following production rules:
Prove that \( \{(ab)^n a \mid n \in \mathbb{N}\} \subseteq \mathcal{L}(G) \).

**Hint:** Start by proving that \( \{(ab)^n aA \mid n \in \mathbb{N}\} \subseteq S(G) \).