1. Answer the following three questions:

   (a) Formally prove that:
   \[ \exists x : X \cdot P \vdash \exists x : X \cdot Q \implies P \]

   (b) Homogeneous relations \( r \) and \( s \), both of type \( X \leftrightarrow X \), are said to commute if \( r \circ s = s \circ r \). Prove that, if \( r \) and \( s \) commute: (i) \( r^{-1} \) and \( s^{-1} \) also commute; (ii) \( r ; s \) and \( r \) also commute.

   (c) Prove that \( \forall n, m : \mathbb{N} \cdot \text{successor}(n) + m = n + \text{successor}(m) \).

2. Answer the following three questions:

   (a) Without assuming soundness and completeness of propositional logic, formally prove that: \( \neg(P \implies Q) \vdash Q \implies P \).

   (b) Prove that, if \( R \cap S \subseteq T \), then \( R \setminus T \subseteq S^c \).

   (c) Consider the following inductive type of numeric expressions over a variable \( x \):
\[
Exp ::= \text{VarX} \\
| \text{Val N} \\
| \text{Mul}(Exp, Exp) \\
| \text{Add}(Exp, Exp)
\]

State the inductive principle for type \(Exp\).

Now consider the following definitions of (i) \(\text{pow}(e)\) which gives the highest power of variable \(x\) in expression \(e\); and (ii) \(\text{diff}(e)\) which differentiates expression \(e\) with respect to variable \(x\):

\[
\begin{align*}
\text{pow}(\text{VarX}) & \overset{\text{def}}{=} 1 \\
\text{pow}(\text{Val n}) & \overset{\text{def}}{=} 0 \\
\text{pow}(\text{Add}(e_1, e_2)) & \overset{\text{def}}{=} \max(\text{pow}(e_1), \text{pow}(e_2)) \\
\text{pow}(\text{Mul}(e_1, e_2)) & \overset{\text{def}}{=} \text{pow}(e_1) + \text{pow}(e_2)
\end{align*}
\]

\[
\begin{align*}
\text{diff}(\text{VarX}) & \overset{\text{def}}{=} \text{Val 1} \\
\text{diff}(\text{Val n}) & \overset{\text{def}}{=} \text{Val 0} \\
\text{diff}(\text{Add}(e_1, e_2)) & \overset{\text{def}}{=} \text{Add}(\text{diff}(e_1), \text{diff}(e_2)) \\
\text{diff}(\text{Mul}(e_1, e_2)) & \overset{\text{def}}{=} \text{Add}(\text{Mul}(e_1, \text{diff}(e_2)), \text{Mul}(\text{diff}(e_1), e_2))
\end{align*}
\]

Assuming standard properties of arithmetic, and using structural induction, prove that \(\forall e : \text{Exp} \cdot \text{pow}(e) \geq \text{pow}(\text{diff}(e))\).