1. Without assuming soundness and completeness of propositional logic, formally prove that: 

\[(P \lor Q) \implies R \iff (P \implies R) \land (Q \implies R)\]

2. Formally prove that:

\[(\exists x : X \cdot \forall y : Y \cdot P) \implies Q \iff \neg Q \implies (\forall x : X \cdot \exists y : Y \cdot \neg P)\]

3. Consider the following definitions of the operators \(\uparrow\) and \(\downarrow\) on natural numbers:

\[
\begin{align*}
0 & \defeq 0 \\
(succ\ n) & \defeq n \uparrow \\
0 & \defeq 0 \\
(succ\ n) & \defeq \text{succ} (n \downarrow)
\end{align*}
\]

Using these definitions, prove that \((n + n) \uparrow = (n + n) \downarrow\).
4. Consider the following inductive type:

\[
Prg ::= \text{Print} \ N \\
| \text{Seq } (Prg \times Prg) \\
| \text{Choice } (Prg \times Prg)
\]

State the inductive principle for type \( Prg \).

Now consider the following definitions:

\[
\begin{align*}
\text{reverse(print } n\text{)} & \overset{\text{def}}{=} \text{print } n \\
\text{reverse(seq(p, q))} & \overset{\text{def}}{=} \text{seq(reverse(q), reverse(p))} \\
\text{reverse(choice(p, q))} & \overset{\text{def}}{=} \text{choice(reverse(p), reverse(q))}
\end{align*}
\]

\[
\begin{align*}
\text{maxsum(print } n\text{)} & \overset{\text{def}}{=} n \\
\text{maxsum(seq } p, q\text{)} & \overset{\text{def}}{=} \text{maxsum}(p) + \text{maxsum}(q) \\
\text{maxsum(choice } p, q\text{)} & \overset{\text{def}}{=} \text{max(maxsum}(p), \text{maxsum}(q))
\end{align*}
\]

assuming standard properties of arithmetic (addition and maximum), and using structural induction, prove that \( \forall p : Prg \cdot \text{maxsum}(p) = \text{maxsum(reverse}(p)) \).