1. Triangular numbers are numbers which are the sum of \( \{1, 2 \ldots n\} \) for some \( n \). Prove that the language \( \Delta = \{ a^m \mid \exists n \cdot m = \sum_{i=0}^{n} i\} \) is not a regular language.

2. An back-and-forth automaton is a normal non-deterministic finite-state automaton, except that it may have multiple initial states. Such an automaton accepts a string \( w \) if two conditions hold: (i) starting from at least one initial state, it may end up in a final state; and (ii) starting from at least one final state, it may end up in an initial state. Note that the first condition is very similar to that of a normal non-deterministic finite state automaton, except that it has a set of initial states it may start off from. Also note that the initial and final state need not be the same in the two conditions.

(a) Formalise the notion of a back-and-forth automaton.

(b) This is not a question. The configuration of such an automaton with states \( Q \) is a pair of sets of states, \( 2^Q \times 2^Q \), to store the states it may reach starting from an initial state, and the set of states it may reach starting from a final state.
(c) Define the set of initial and final configurations. Recall that the first set corresponds to the set of states reachable from some initial state, while the second corresponds to the states reachable from some final state.

(d) Define \( c \xrightarrow{a} c' \), meaning that configuration \( c \) goes to configuration \( c' \) reading symbol (not string) \( a \).

(e) Define \( c \xrightarrow{w} c' \), meaning that configuration \( c \) can go to configuration \( c' \) consuming string \( w \).

(f) Hence or otherwise define the language accepted by a back-and-forth automaton.