1. Prove that Unextendible Clique (UCLI) is NP-Complete, where UCLI is defined as: Given a graph \( G \) and a positive integer \( k \), does \( G \) have a clique of size \( k \), which cannot be extended into a clique of size \( k + 1 \) by adding one node? **Hint:** A graph with \( n \) nodes has a clique of size \( k \) if and only if it has a unextendible-sized clique of size between \( k \) and \( n \).

2. Prove that the language \( I = \{ a^n b^m \mid n > m \} \) is not a regular language. Prove also that \( II = \{ a^n b^m \mid n < m \} \) is not a regular language. Hence or otherwise prove that \( III = \{ a^n b^m \mid n \neq m \} \) is not regular.

3. A loopy automaton is a normal non-deterministic finite-state automaton, except that a string is accepted, if when following it starting from the initial state, at the end one may end up in a final state going through a loop (repeating at least one state) in the process.

   (a) Formalise the notion of a loopy automaton.

   (b) Formalise the *configuration* of these automata. Hint: You have to remember the states which you have already visited at least once, and whether you have already taken a loop.

   (c) Define \( C_0 \), the initial configuration, and the set \( C_F \), the set of final configurations.
(d) Define $C \xrightarrow{a} C'$, meaning that configuration $C'$ can be reached from configuration $C$ reading symbol (not string) $a$.

(e) Define $C \Rightarrow C'$, meaning that configuration $C'$ can be reached from configuration $C$ with string $s$.

(f) Hence or otherwise define the language accepted by an loopy automaton.