1. Prove that 2-CLIQUES is NP-Complete, where 2-CLIQUES is defined as: Given a graph $G$ and number $k$, does $G$ contain two disjoint cliques, each of size $k$?

2. (a) Consider the language $L_a = \{ w\psi_a(w) \mid w \in \Sigma^* \}$, where $\psi_a(w)$ is defined to be $w$, but without any instances of symbol $a$ (you may assume that $a \in \Sigma$).

   i. Formally define $\psi_a(w)$.
   ii. Prove that $L_{\{a,b\}}$ is not a regular language.
   iii. Prove that $L_{\{a,b,c\}}$ is not a context-free language.
   iv. Show that $L_{\{a\}}$ is a regular language.

   (b) An $\alpha$-coloured state automaton (where $\alpha$ is a fixed finite set of colours) is a normal non-deterministic finite-state automaton, except that it has no final states, and every state is of exactly one colour from $\alpha$. A string is accepted by such an automaton if, starting from the initial state, and following the string, the path followed will pass through at least one state of each colour in $\alpha$.

   i. Formalise the notion of an $\alpha$-coloured state automaton.
   ii. Formalise the configuration of these automata. **Hint:** The configuration must also store the set of colours already encountered.
iii. Define $C_0$, the initial configuration, and the set $C_F$, the set of final configurations.

iv. Define $C \xrightarrow{a} C'$, meaning that configuration $C'$ can be reached from configuration $C$ reading symbol (not string) $a$.

v. Define $C \xrightarrow{\delta} C'$, meaning that configuration $C'$ can be reached from configuration $C$ with string $s$.

vi. Hence or otherwise define the language accepted by an $\alpha$-coloured state automaton.

3. (a) A composite number is a natural number which has at least one factor other than itself and 1. Consider the language $C$, which is defined as $C = \{ w^c \mid c \text{ is a composite number} \}$. Prove that $C$ is not a regular language.

(b) A prime automaton is a normal non-deterministic finite state automaton, except that a string is accepted if the path followed goes through a final state a prime number of times.

i. Formalise the notion of a prime automaton.

ii. Formalise the configuration of these automata. **Hint:** The configuration must also store the number of times final states have already been traversed.

iii. Define $C_0$, the initial configuration, and the set $C_F$, the set of final configurations.

iv. Define $C \xrightarrow{a} C'$, meaning that configuration $C'$ can be reached from configuration $C$ reading symbol (not string) $a$.

v. Define $C \xrightarrow{\delta} C'$, meaning that configuration $C'$ can be reached from configuration $C$ with string $s$.

vi. Hence or otherwise define the language accepted by a prime automaton.

vii. If $L_r$ is the class of regular languages and $L_p$ is the class of languages accepted by prime automata, prove that $L_p \not\subseteq L_r$. 

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**end of paper**