1. A universal non-deterministic finite state automaton (\(\forall\text{-NPDA}\)) is identical to a normal non-deterministic finite state automaton (\(\text{NPDA}\)), except that it accepts a string \(w\) if all computation paths starting from the initial state, following string \(w\), lead to a final state.

(a) Formalise the notion of a \(\forall\text{-NPDA}\) including what we mean by the configuration of such a machine, and the language accepted by a \(\forall\text{-NPDA}\).

[15 marks]

(b) Prove that every regular language is accepted by a \(\forall\text{-NPDA}\).

\textbf{Hint:} Every deterministic finite state automaton is a \(\forall\text{-NPDA}\).

[15 marks]

(c) Show that every language accepted by a \(\forall\text{-NPDA}\) is regular.

\textbf{Hint:} Consider the determinisation of a \(\forall\text{-NPDA}\).

[20 marks]

2. Consider the language of strings with an equal number of 0s and 1s:
\[ E = \{ w \in \{0,1\}^* \mid \sharp_0(w) = \sharp_1(w) \} \]

(a) Show that \(E\) is a context-free language.

[15 marks]
(b) Prove that $E$ is not a regular language. \[ 15 \text{ marks} \]

(c) Prove that $E^* = E$.

\textbf{Hint:} Open up the definition of $E^*$ and use induction on the number of iterations. \[ 20 \text{ marks} \]

3. (a) Consider the following problem:

\textsc{Loop (LP)}

\textbf{Instance:} A deterministic finite state automaton $M$ and a state $q$.

\textbf{Question:} Is there a computation loop in $M$ which passes through state $q$?

Show that $LP \in P$. \[ 20 \text{ marks} \]

(b) Consider the following problem:

\textsc{Bin Packing (BP)}

\textbf{Instance:} A set of items of size $i_1 \ldots i_m$ and a set of $n$ knapsacks, each of capacity $k$.

\textbf{Question:} Can the items be fitted into the $n$ knapsacks without exceeding the maximum capacity?

Show that $BP \in NP$. \[ 20 \text{ marks} \]

(c) $NPSPACE$ is the class of problems that can be solved in polynomial space on a non-deterministic Turing Machine. Argue that $NP \subseteq NPSPACE$. \[ 10 \text{ marks} \]

4. (a) Consider the problem \textit{Repeat:} Given a Deterministic Turing Machine $M$, input $x$ and set of states $Q$, does $M$ with input $x$ pass infinitely often through states in $Q$ (possibly also going through other states not in $Q$)?
Show that *Repeat* is undecidable.

**Hint:** Use reducibility.  

[30 marks]

(b) Now consider a similar problem for finite state automata: Given a deterministic finite state automaton and set of states $Q$, is there a loop which passes through state $q$? Show that this is decidable.  

[20 marks]