An Introduction to Fuzzy Logic

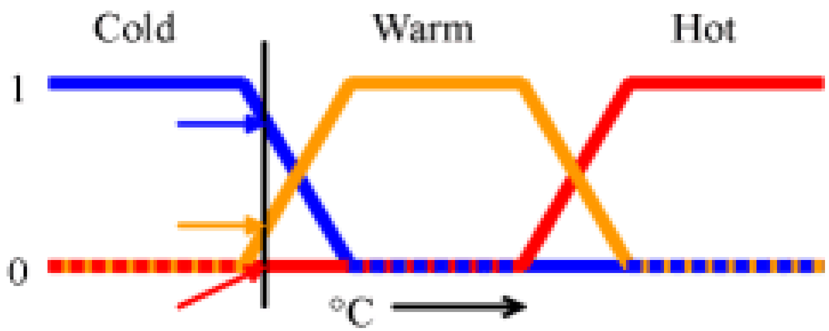
2007, Kristian Guillaumier

Introduction

- Based on fuzzy set theory: true, false, 'in between' – approximate reasoning.
- Concept of a degree of truth: truth is defined as membership in some vaguely defined set (not probability based – no element of randomness).
- Wikipedia example:
- Bob is in a house with two adjacent rooms: the kitchen and the dining room. In many cases, Bob's status within the set of things "in the kitchen" is completely plain: he's either "in the kitchen" or "not in the kitchen". What about when Bob stands in the doorway? He may be considered "partially in the kitchen". Quantifying this partial state yields a fuzzy set membership. With only his big toe in the dining room, we might say Bob is 99% "in the kitchen" and 1% "in the dining room", for instance. No event (like a coin toss) will resolve Bob to being completely "in the kitchen" or "not in the kitchen", as long as he's standing in that doorway.

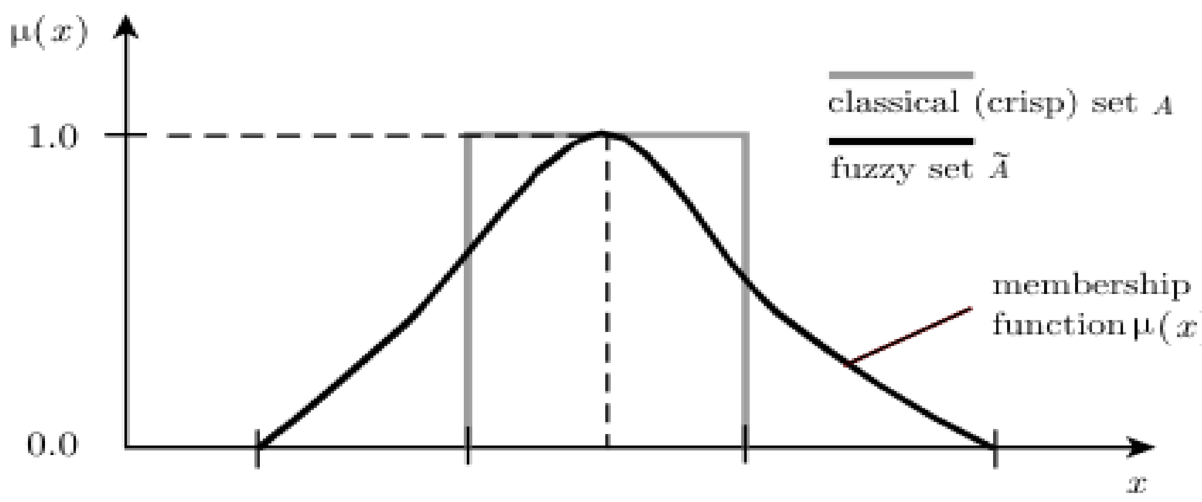
Set Membership

- Membership values range from 1..0.
  - As opposed to Boolean, 'Crisp' logic.
- This caters for instances of 'sort of', 'almost', 'very', etc...
- Introduced by Lotfi Zadeh (1965).
- Temperature example from Wikipedia:
  - Functions that define cold, warm and hot.
  - Each membership function maps temperatures to a value 1..0.



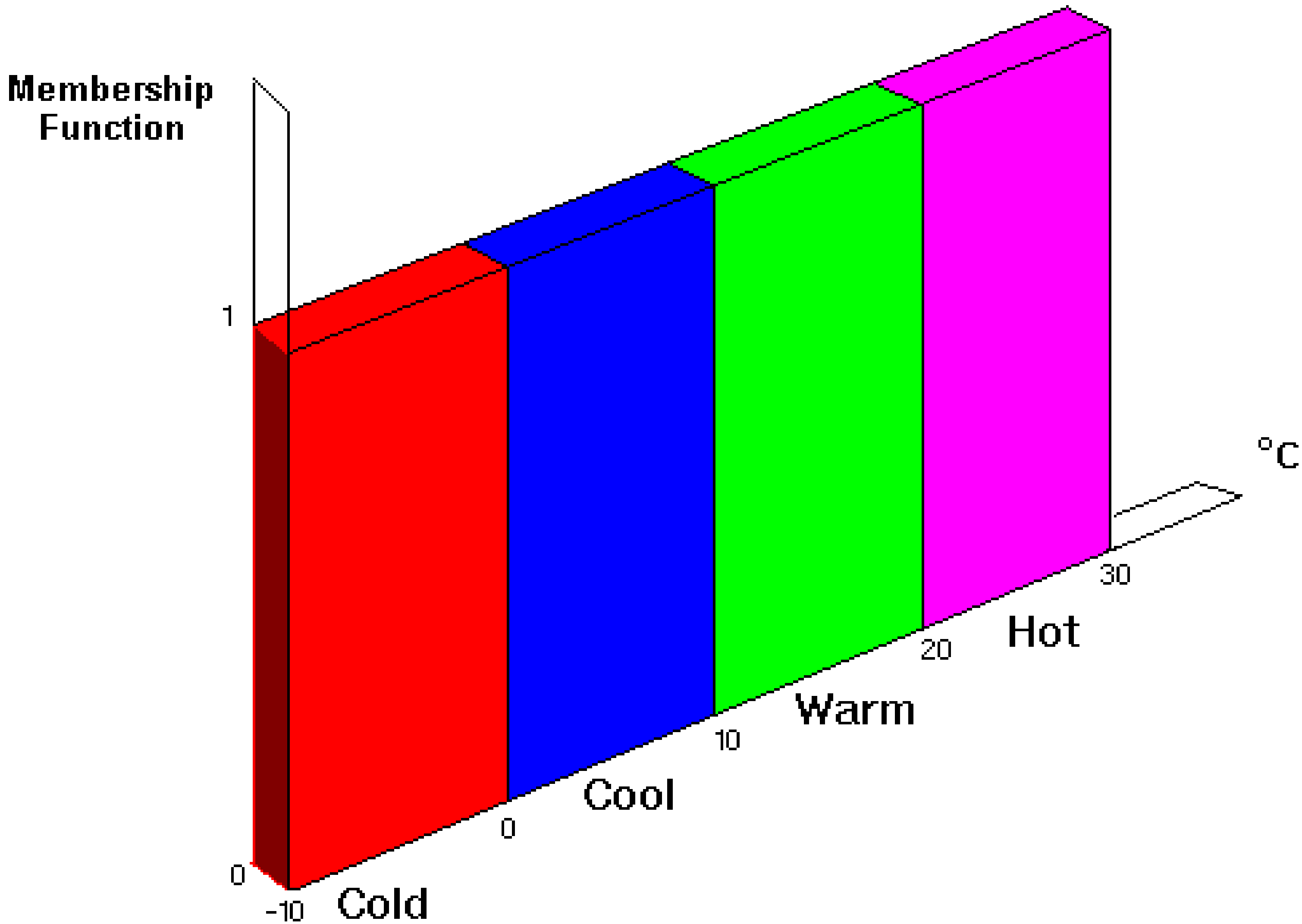
Set Membership

- Based on a membership function mu --> [0,1].
- mu is standard nomenclature for a fuzzy truth value.
- Image from Wikipedia:



Set Membership

from http://www.doc.ic.ac.uk/~nd/surprise_96/journal/vol2/jp6/article2.html



Set Membership

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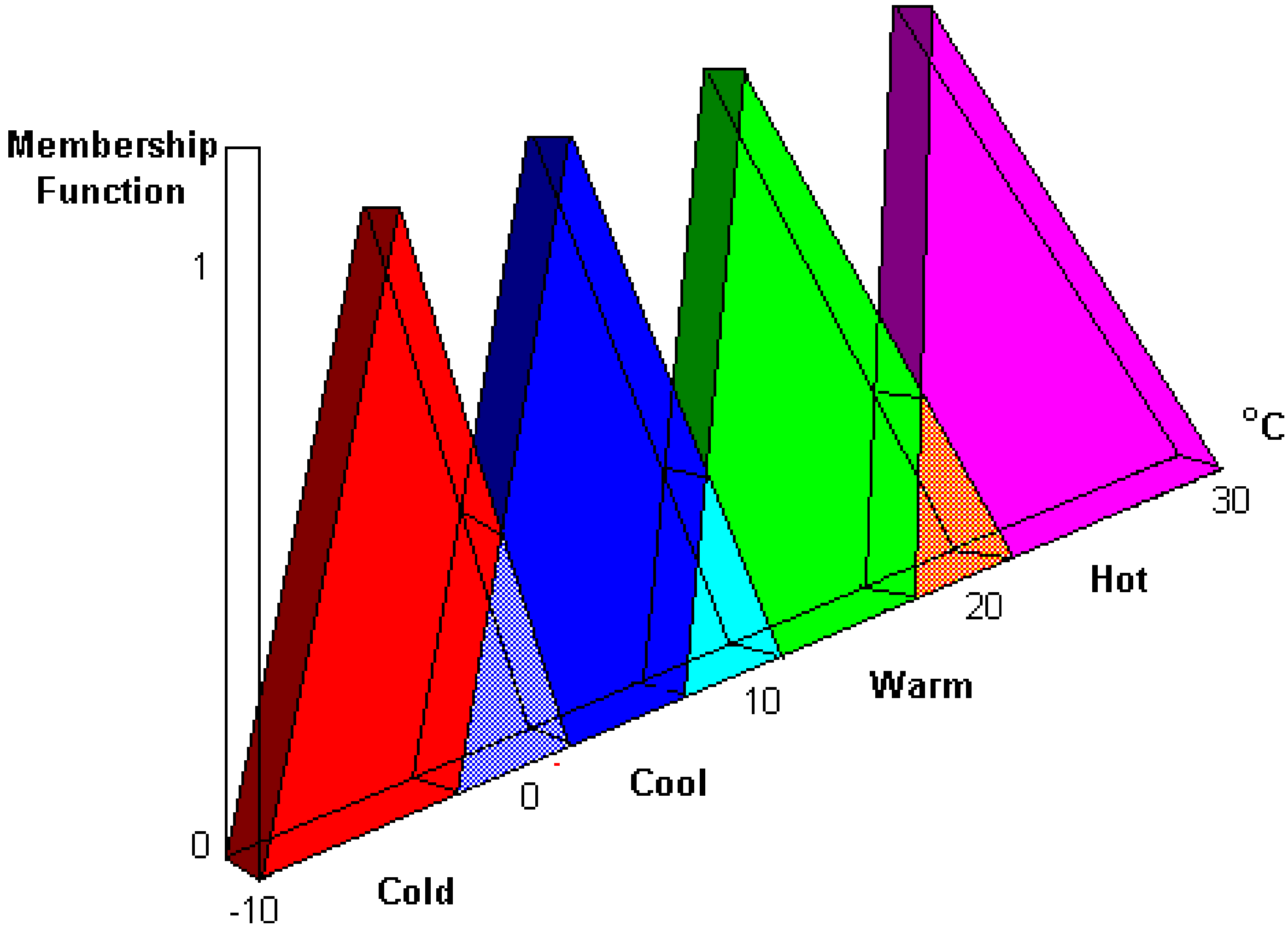


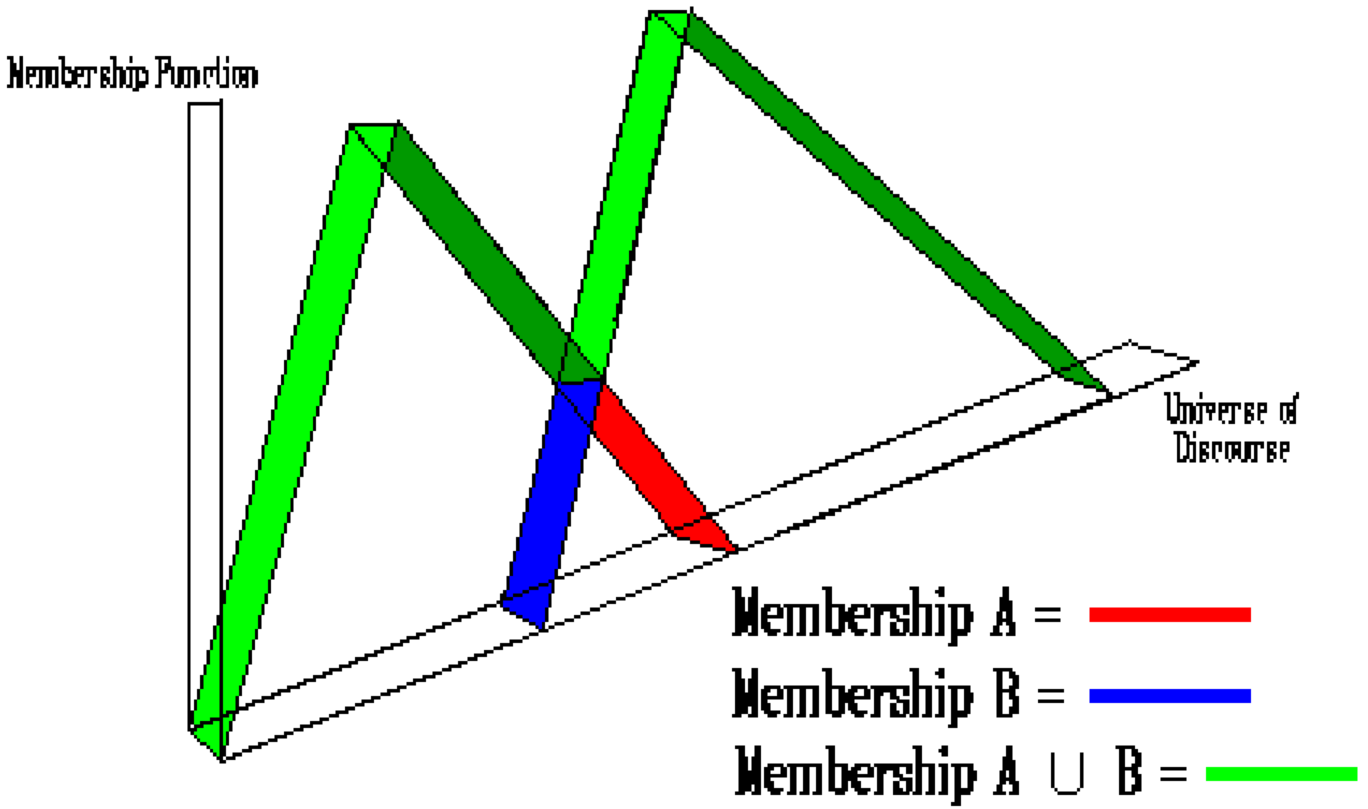
Set Membership

- Based on IF..THEN rules.
- Note there is no ELSE part:
  - Something can be both slightly warm and slightly cold at the same time.
- Membership functions can be triangular, bell-shaped, trapezoidal...

An Example Membership Function

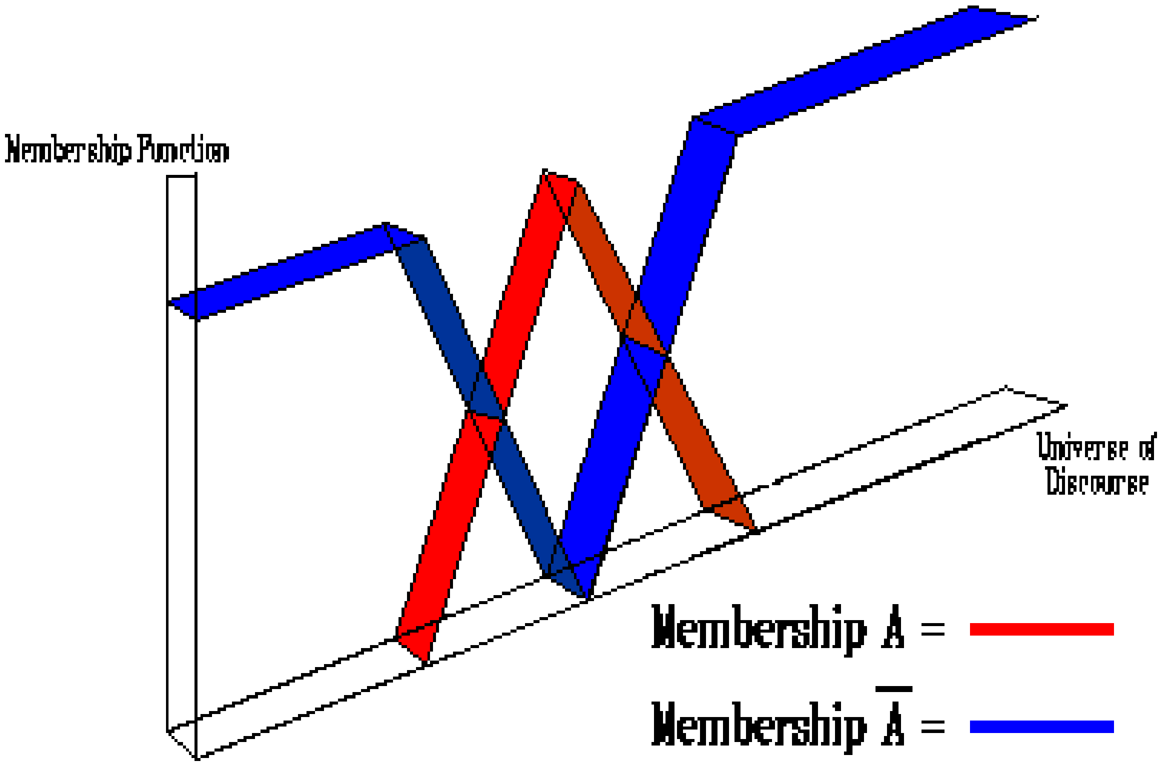
- IsTall(x) =
  - 0; x <= 1.5.
  - 1; x >= 2.0.
  - (x – 1.5) / (2.0 – 1.5); x > 1.5, x < 2.0.

Fuzzy Operators – OR



Fuzzy Operators - Complement/NOT

$$\mu_{\overline{A}} = 1 - \mu_A$$



Fuzzy Operators

- OR:
  - A OR B (x) = MAX(A(x), B(x)) // Union.
  - ~A(x) = 1-A(x).
  - A AND B (x) = MIN(A(x), B(x)) // Intersection.

Algebraic Laws Still Apply

- De Morgans.
- Associativity:
  - (A AND B) AND C = A AND (B AND C).
- CommutativityL
  - A AND B = B AND A.
- Distributivity:
  - A AND (B OR C) = (A AND B) OR (A AND C).
- Etc...

Hedges

- Hedges are linguistic modifiers on membership functions.
- Consider the membership function Is Tall(x).
- A hedge would be:
  - Is ***Very*** Tall(x).
  - Is ***Sort Of*** Tall(x).
  - Is ***Quite*** Tall(x).
  - Is ***Slightly*** Tall(x).
  - Is ***Somewhat*** Tall(x).

Mathematical Representation of Hedges

- Consider a membership function mu(x):
  - A little = $[mu(x)]^{1.3}$
  - Slightly = $[mu(x)]^{1.7}$
  - Very = $[mu(x)]^2$
  - Extremely = $[mu(x)]^3$
  - More or Less, Somewhat = SQRT[mu(x)]

Mathematical Representation of Hedges

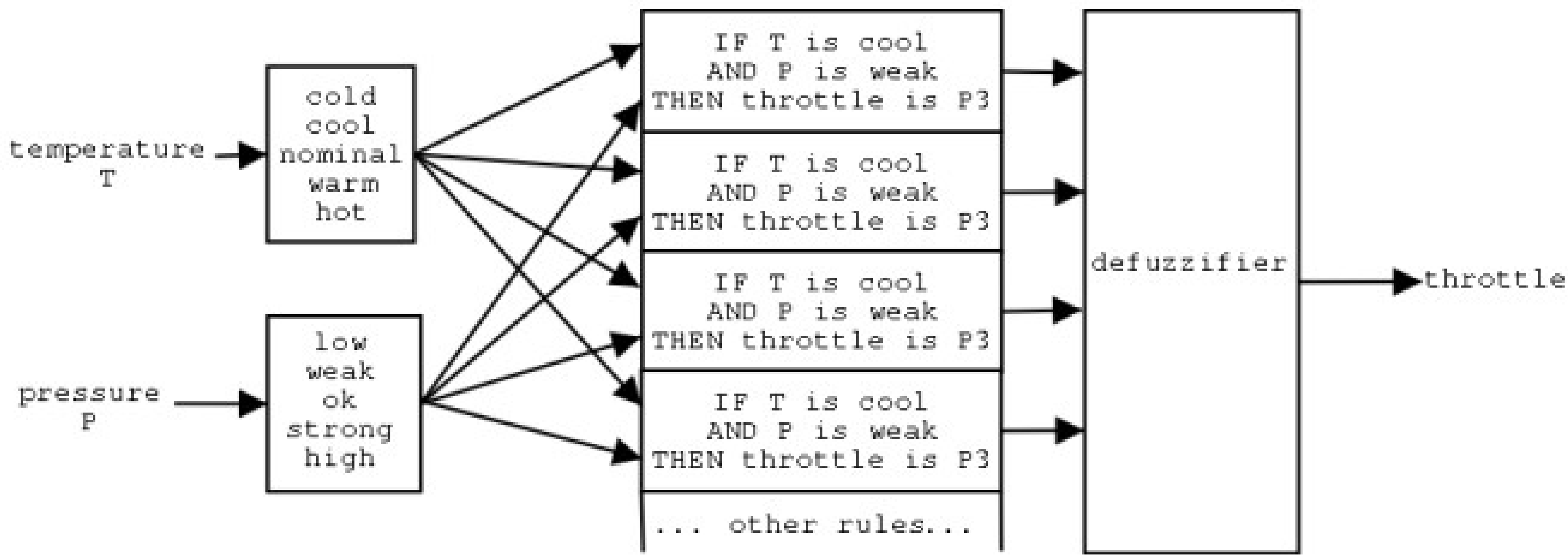
- From the previous membership function for IsTall, consider IsTall(John) = IsTall(1.75).
- This gives a fuzzy truth value of 0.8333.
- IsSomewhatTall = SQRT(0.8333) = 0.913.
- IsVeryTall = 0.8333^2 = 0.694.
- Etc...

Fuzzy Control

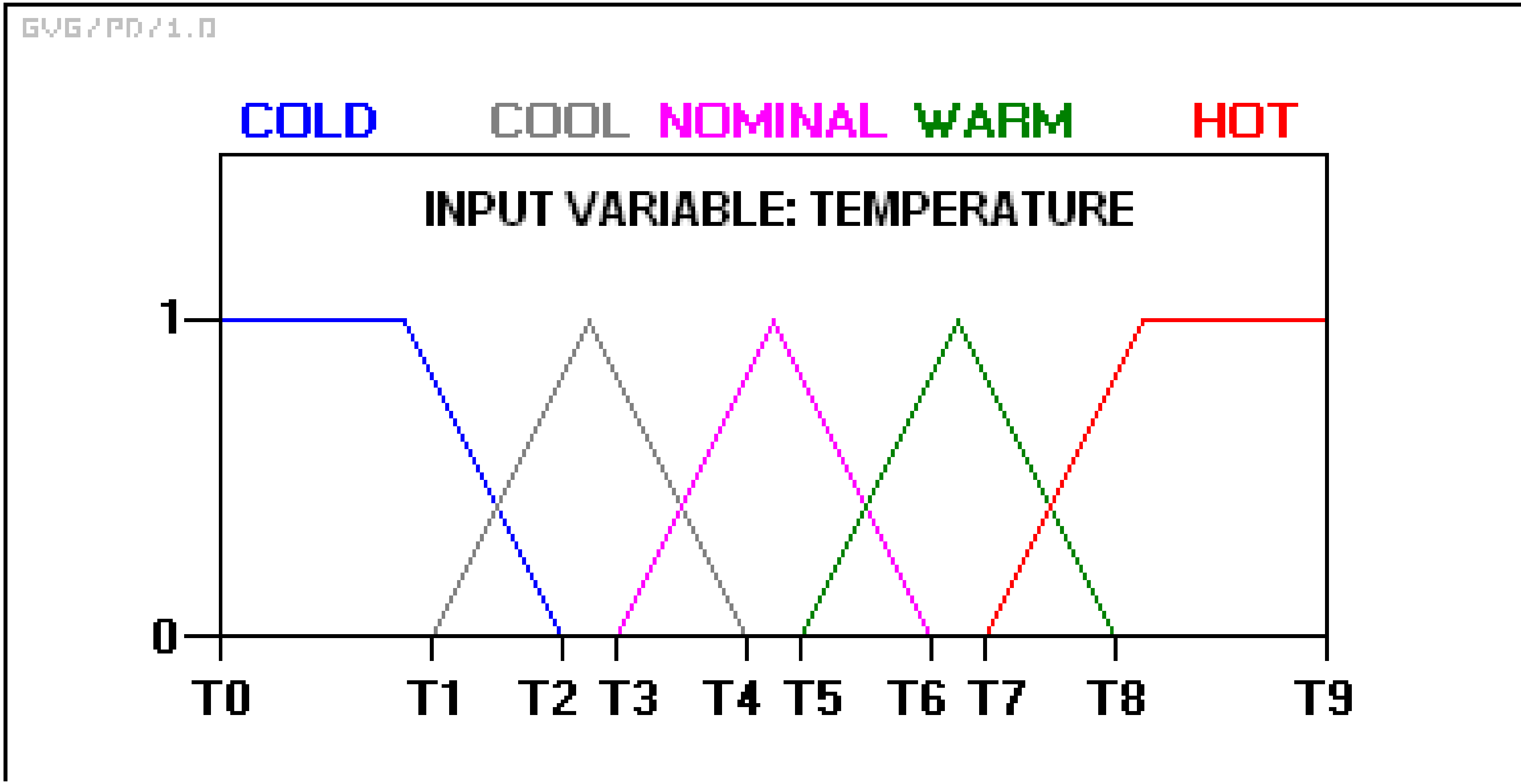
- Inputs from a sensor (e.g. Temperature, speed).
  - Possibility of ON/OFF (crisp 1, 0) inputs.
- Outputs to an actuator (e.g. Pressure).
- Wikipedia example:
  - IF BrakeTemperatureIsWarm AND SpeedIsNotVeryFast THEN BrakePressureIsSlightlyDecreased.
- There may be many fuzzy rules in a system (fuzzy rule set) and each rule may have conjunctives (AND, OR).

Defuzzification

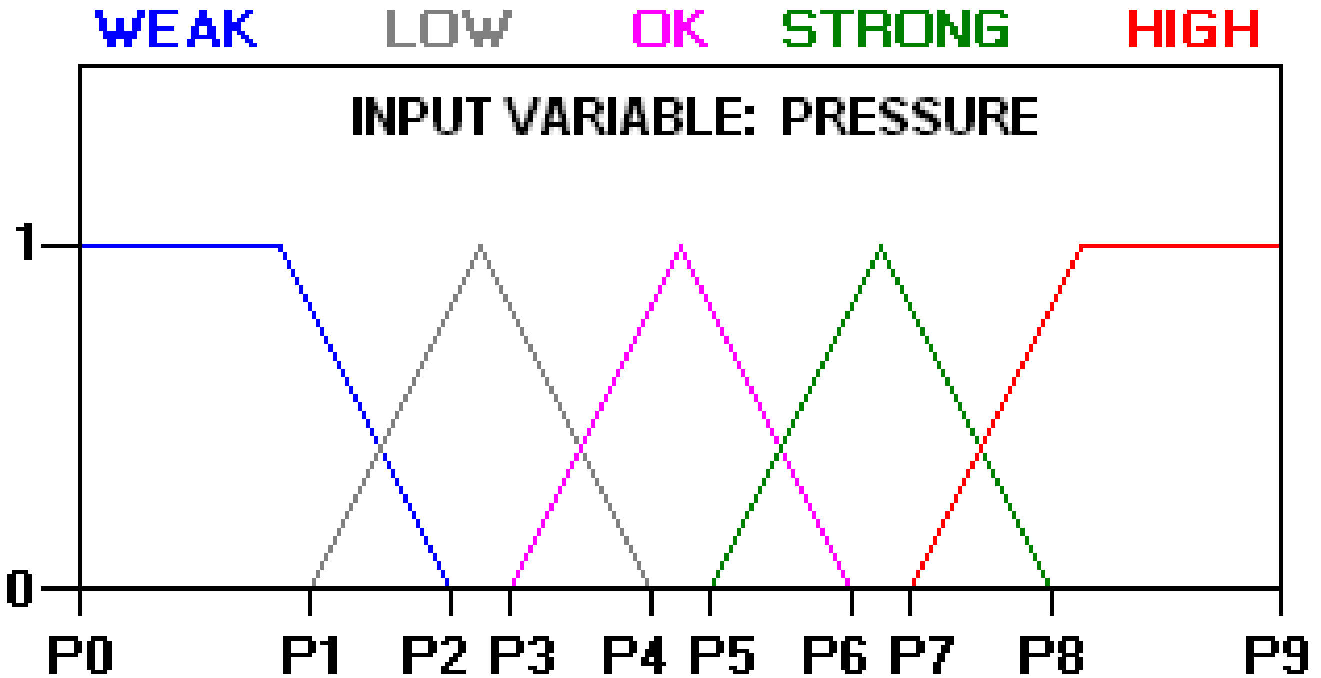
From http://en.wikipedia.org/wiki/Fuzzy_control_system



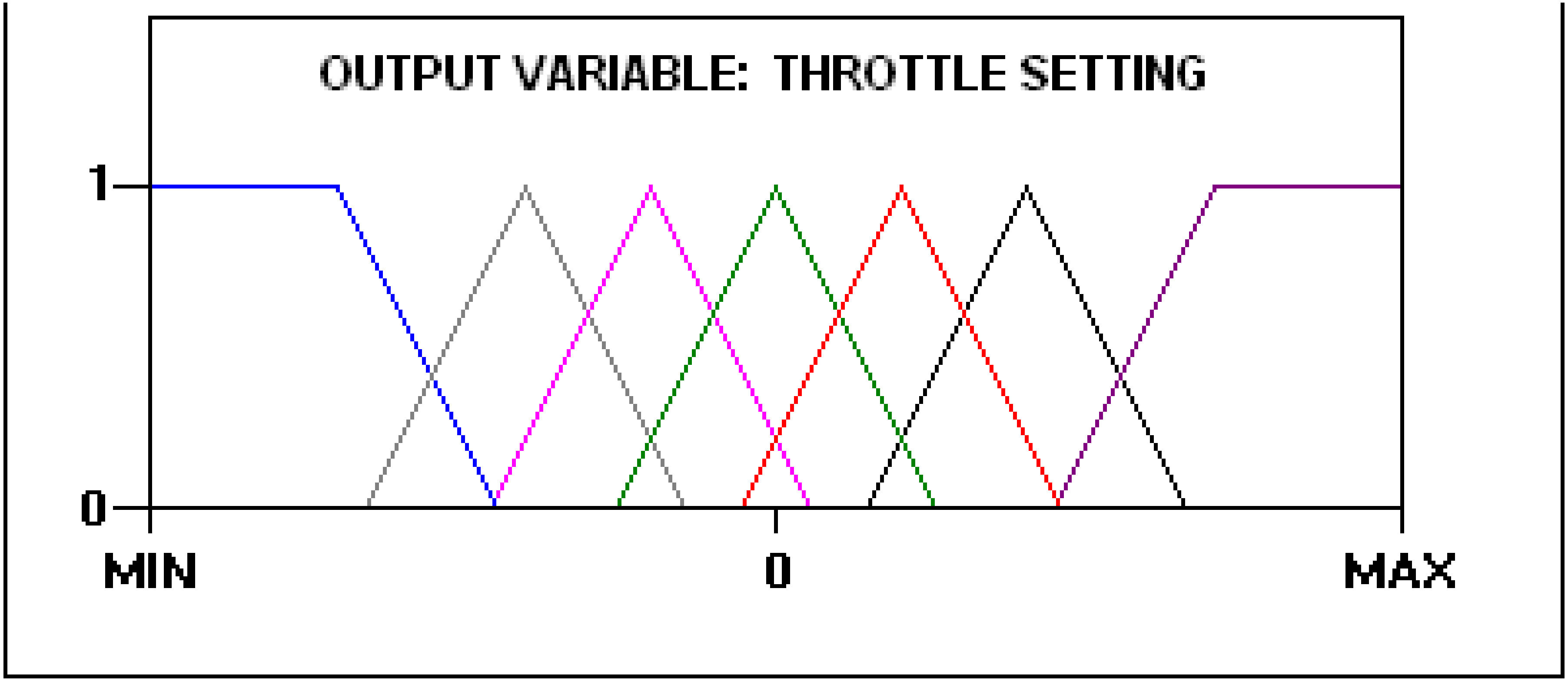
Example (from Wikipedia)



Example


WEAK
LOW
OK
STRONG
HIGH
INPUT VARIABLE: PRESSURE
1
0
P0
P1
P2
P3
P4
P5
P6
P7
P8
P9

Example



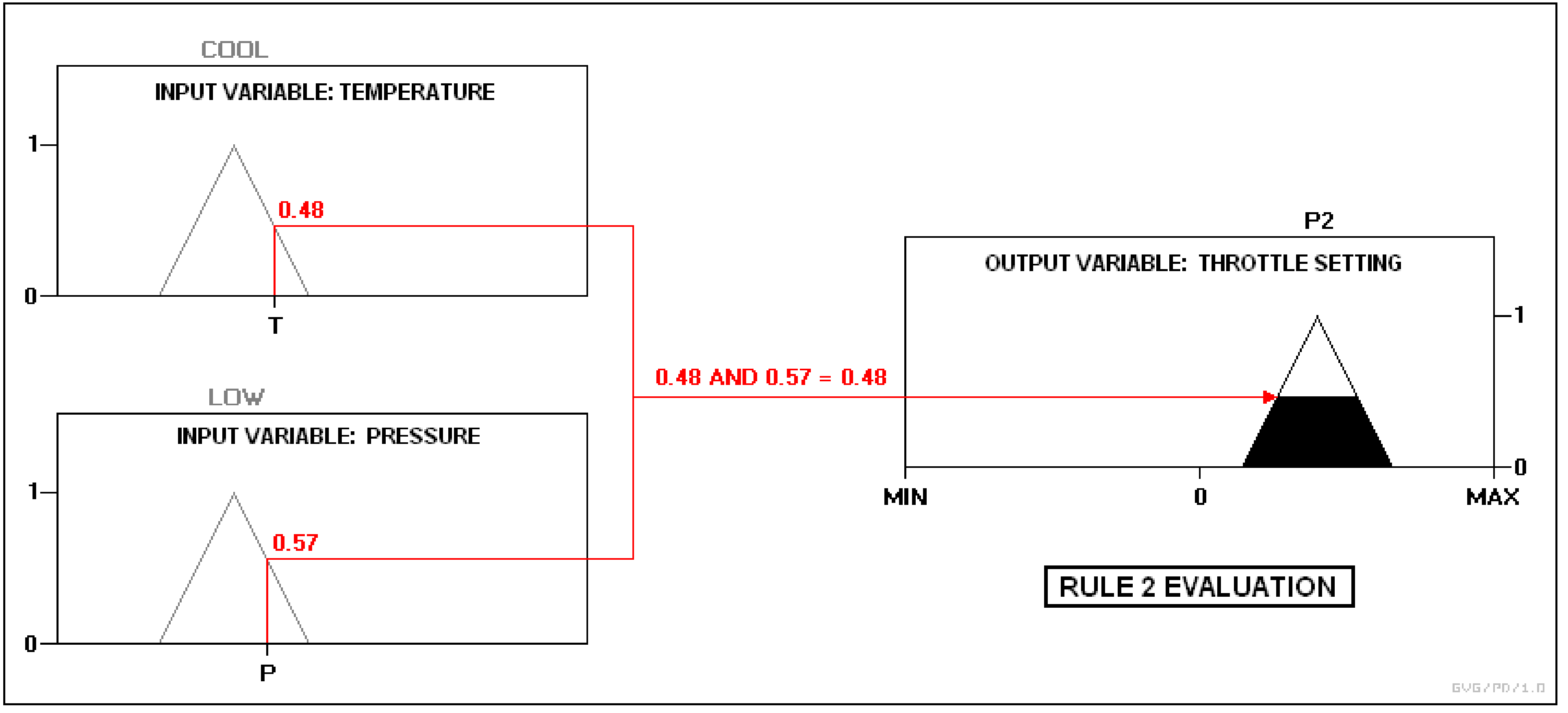
Example Output Values

- N3 = Large Negative
- N2 = Medium Negative
- N1 = Small Negative
- Z = Zero
- P1 = Small Positive
- P2 = Medium Positive
- P3 = Large Positive

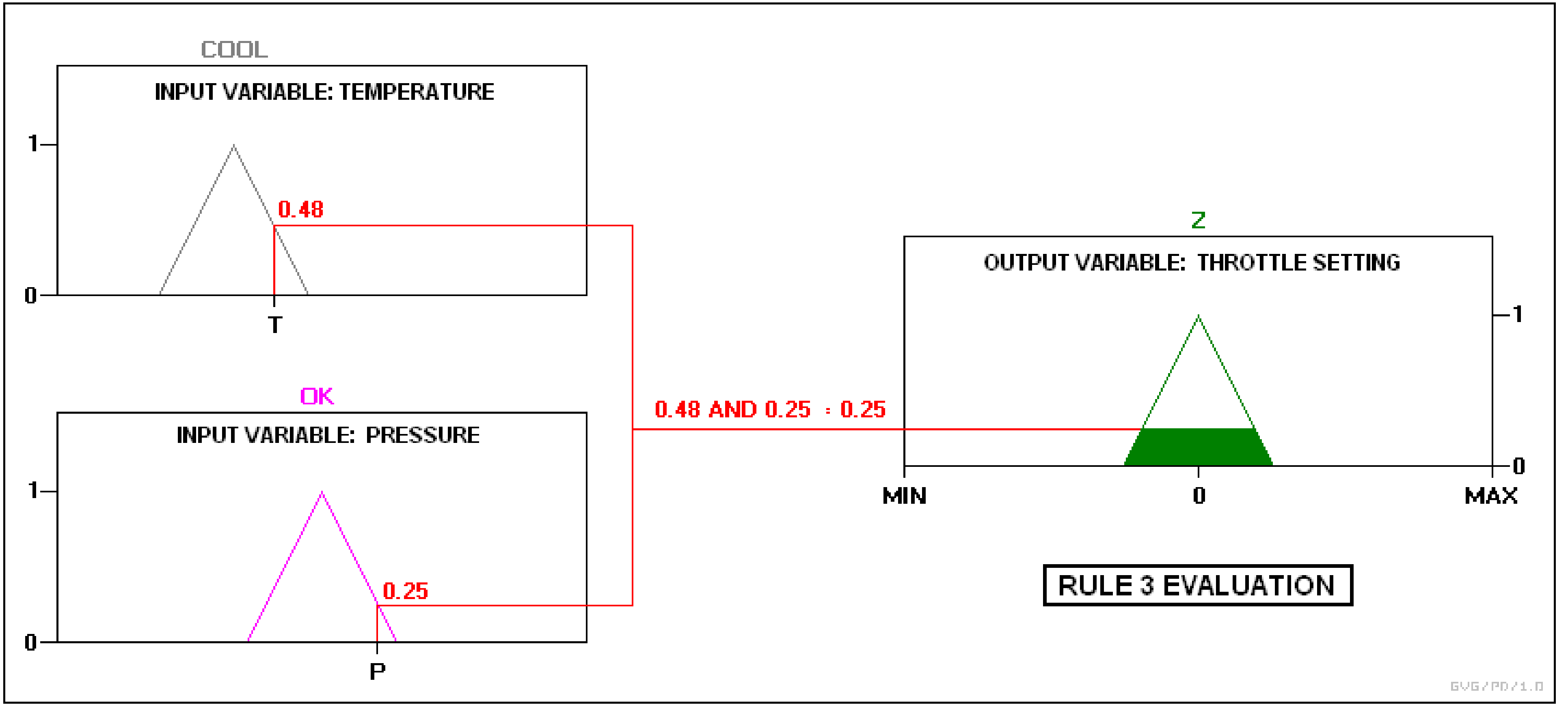
Example Rules

- IF temperature IS cool AND pressure IS weak, THEN throttle is P3.
- IF temperature IS cool AND pressure IS low, THEN throttle is P2.
- IF temperature IS cool AND pressure IS ok, THEN throttle is Z.
- IF temperature IS cool AND pressure IS strong, THEN throttle is N2.
- Assume that the temperature is in the cool state and the pressure is in the low and ok states. Rules 2 and 3 will fire.

Evaluation


COOL
INPUT VARIABLE: TEMPERATURE
1
0
0.48
T
LOW
INPUT VARIABLE: PRESSURE
1
0
0.57
P
0.48 AND 0.57 = 0.48
P2
OUTPUT VARIABLE: THROTTLE SETTING
1
0
MIN
0
MAX
RULE 2 EVALUATION
GVG/PD/1.0

Evaluation



COOL
INPUT VARIABLE: TEMPERATURE
1
0.48
0
T
OK
INPUT VARIABLE: PRESSURE
1
0.25
0
P
0.48 AND 0.25 : 0.25
Z
OUTPUT VARIABLE: THROTTLE SETTING
1
0
MIN
0
MAX
RULE 3 EVALUATION
GVG/PD/1.0

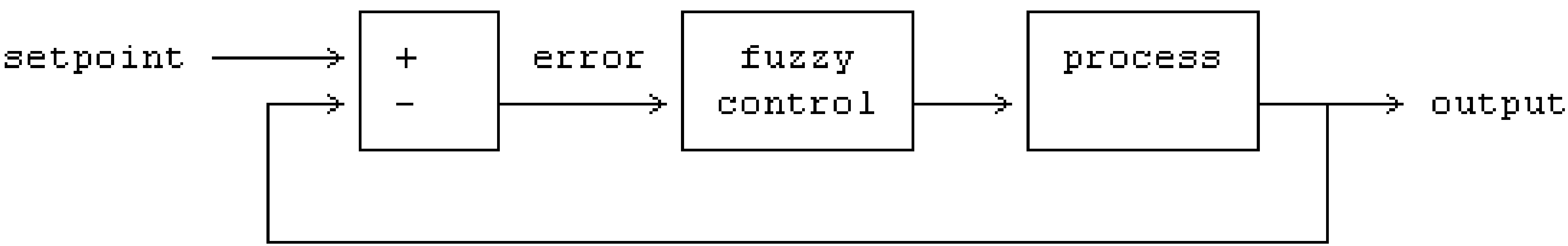
Evaluation

- So, by rule 2, the pressure will be P2 (medium positive) with at truth value of 0.48.
- And by rule 3, the pressure will be Z (zero) with a truth value of 0.25.
- We now have to defuzzify the 2 rule outputs into one output value.
- Defuzzification can be done by either:
  - Choosing the maximum (P2 because it is 0.48, versus 0.25 for Z). This is not often used.
  - Choosing the centroid.

Centroid Evaluation

```
   |          Z      P2
1 -+          *       *
   |         * *     * *
   |        *   *   *   *
   |       *     * *     *
   |      *       222222222
   |     *       22222222222
   |    333333332222222222222
   +---33333333222222222222222-->
                    ^
                  +150
```

Controllers with Feedback



Controllers with Feedback

- The system will generate:
  - Error (e).
  - Change in Error (delta).
- A rule base will exist that creates an output (repair) for the Error and Change In Error. E.g.:
  - If e = Z AND delta = Z THEN output = Z
  - If e = Z AND delta = SP THEN output = SN
  - If e = SN AND delta = SN THEN output = LP
  - If e = LP OR delta = LP THEN output = LN

Error/Delta Table

	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
mu(LP)	0	0	0	0	0	0	0.3	0.7	1
mu(SP)	0	0	0	0	0.3	0.7	1	0.7	0.3
mu(ZE)	0	0	0.3	0.7	1	0.7	0.3	0	0
mu(SN)	0.3	0.7	1	0.7	0.3	0	0	0	0
mu(LN)	1	0.7	0.3	0	0	0	0	0	0

Controllers with Feedback

- Suppose that at a point in time we have e = 0.25, and delta = 0.5.
- By rule 1, mu(1) = MIN(0.7, 0.3) = 0.3, output = 0.
- By rule 2, mu(2) = MIN(0.7, 1) = 0.7, output = -0.25.
- By rule 3, mu(3) = MIN(0, 0) = 0, output = 0.75.
- By rule 4, mu(4) = MAX(0, 0.3) = 0, output = -1.

Computing the Centroid

- For each rule x:
  - SUM(mu(x) * output(x))

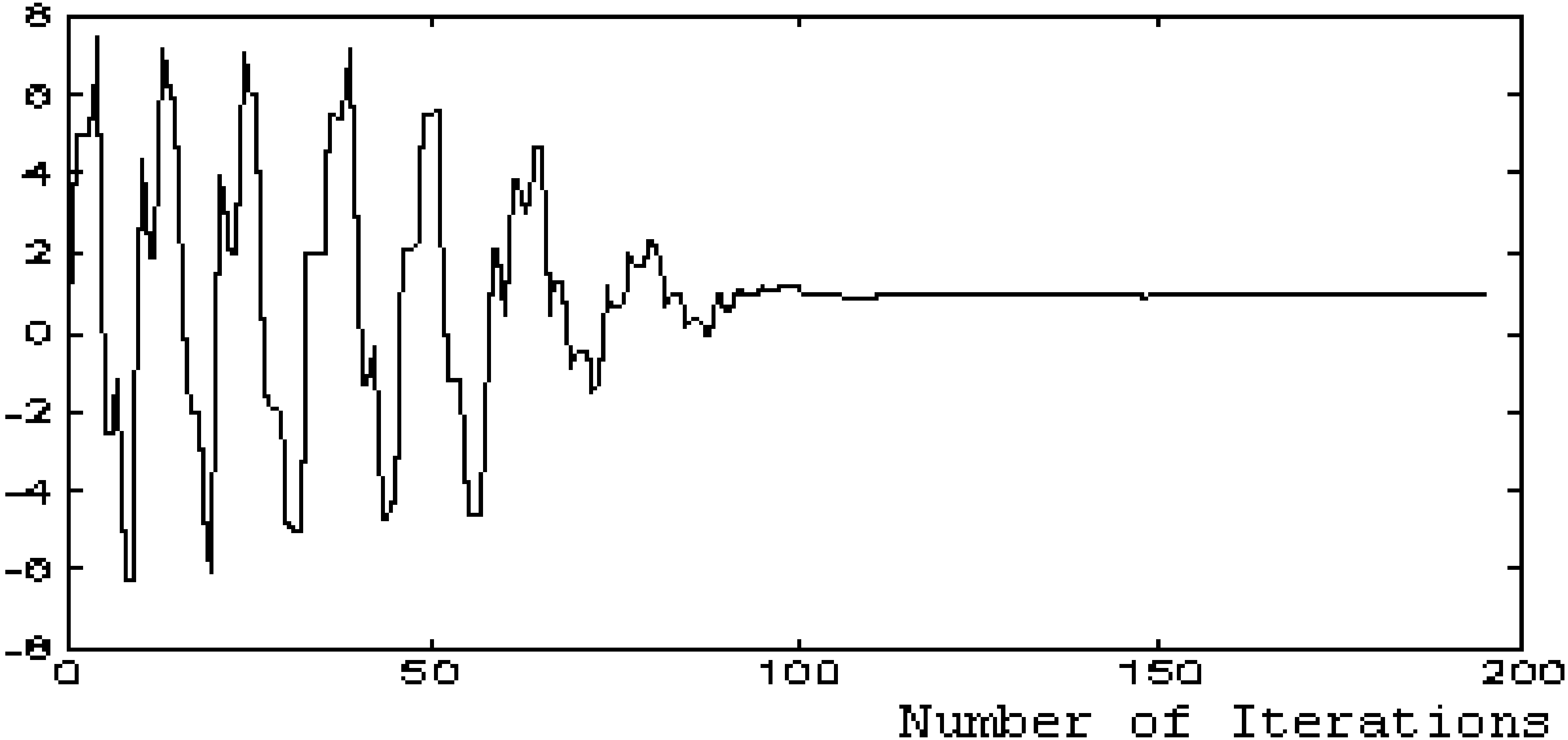
  Divided by:
  - SUM(mu(x))
- In this case:

$$\frac{(0.3*0) + (0.7*-0.5) + (0*0.75) + (0.3*-1)}{0.3 + 0.7 + 0 + 0.3}$$

  - = -0.5

Fluctuations

From http://www.ici.ro/ici/revista/sic1997_2/



Advantages of Fuzzy Systems

- Do not require a mathematical model.
- Can fuzzify an expert system.
- Simple engine.