This assignment is worth 8% of the final mark of the declarative programming course. A single file, with the code and inline documentation is to be submitted online by midnight of Sunday 24th March 2007. You can find the online submission system on Joseph Cordina's website under the Functional Programming link. Hopefully you have picked up your registration ticket by the time you are reading this. You are also to hand in a signed copy of the plagiarism form to Mr Vincent Sammut by Monday 25th March 2007. Without such a form, your assignment will not be marked. Just in case, the online submission server dies, you can send your assignment by e-mail to gordon.pace(at)um.edu.mt. Assignments handed in late will be marked to zero.

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http://www.cs.um.edu.mt/resources/plagiarism/

Regular Expressions.

The aim of this assignment is to build a regular expression parser in Haskell. Regular expressions are frequently used in programming and text editors to search for and manipulate text. You will be seeing more of regular expressions in the B.Sc. IT course.

Regular expressions take one of the following forms:

- \texttt{c} (where \texttt{c} is any character) matches only with the specified character.
- \texttt{?} matches with any single character.
- \texttt{e ; f} (where both \texttt{e} and \texttt{f} are themselves regular expressions) matches with a string which matches regular expression \texttt{e}, followed by regular expression \texttt{f}.
- \texttt{e + f} (where both \texttt{e} and \texttt{f} are themselves regular expressions) matches with a string which either matches regular expression \texttt{e}, or regular expression \texttt{f}.
• $e^*$ (where $e$ is itself regular expressions) matches any number of repetitions (possibly zero) of regular expression $e$.

• # matches the empty string.

The concept can be better explained via examples:

• The regular expression $a ; b$ matches with the string $ab$.

• The regular expression $a ; (b + c)^* ; a$ matches with strings which start and finish with an $a$, and with any number of $b$s and $c$s in between, such as $abcbbcaca$ or $aba$, or even $aa$.

• To illustrate the behaviour of $?$, consider the regular expression $a ; ?^* ; a$ which matches any string that starts with an $a$, and consumes all the characters afterwards, such as $abcabcad$ or $aringa$, or even $aa$.

• The regular expression $a ; (bc + cb + #) ; d$ would match a string starting with an $a$, ending with a $d$, and with $bc$, $cb$ or nothing in between (i.e. $abcd$, $acbd$, $ad$).

• A digit (single number) can be described by $0+1+2+3+4+5+6+7+8+9$. Let's call this regular expression $\text{DIGIT}$.

• Positive numbers can be described by $\text{DIGIT} ; \text{DIGIT}^*$. Note that we write this to ensure that it contains at least one digit. Let's call this $\text{NUM}$.

• Floating point numbers (with a decimal point) can now be described by $\text{NUM} ; . ; \text{NUM}$.

• Finally, valid variable names in various languages contain only alphabetic symbols, digits and the underscore symbol, with the additional constraint that they do not start with a digit. If the regular expression matching an alphabetic symbol $a+b+...+z$ is named $\text{ALPHA}$, we can define variable names as $\text{ALPHA} ; (\text{ALPHA} + \text{DIGIT} + _)^*$

The most straightforward use of a regular expression is to try to match it with the prefix of a given string. Thus, for example, trying to match the regular expression $a ; (b + c)^* ; a$ with the string "abccbabbbhellobbc" would match the prefix "abccba" leaving "bbhellobbc" unparsed. Therefore, parsing a string with respect to a given regular expression in such a manner would return a pair of strings. However, as you know, life is not so simple.

Firstly, parsing does not always succeed. For example, regular expression $a;b^*;c$ does not match string "abbbbdccc". Therefore, the result of parsing can be either a pair of strings (as already explained), or an error indicating no correct match. One way of encoding this in Haskell is to encode the result in the type $\text{Maybe (String, String)}$. And there's yet another complication...
Consider parsing the string "bcdefg" with regular expression \((b;c;d) + (b;c)\). This can match in two different ways — either matching "bcd", leaving "ef" unparsed, or matching "bc", leaving def unparsed. Similarly, matching a* to string "aaaha" can match in multiple ways: matching with the empty string "", leaving "aaaha" or matching "a", leaving "aaha" or matching "aa", leaving "aha", or even matching "aaa", leaving "ha". To simplify matters, in both cases we take the ‘correct’ parse to be the longest matching string. Since we apply this rule with every + and * operator, and we parse from left to right, this sometimes leads to anomalous results. For example, matching regular expression a*;a;b to string aab will not match since the a* will greedily match all the initial a, leaving none to match the second part of the regular expression. Do not worry too much about this — you’ll be shown how to solve this when the time is right.

In Haskell we can describe regular expressions using an abstract datatype:

```haskell
data RegExp =
    | Nil -- corresponds to #
    | Any -- corresponds to ?
    | Chr Char -- corresponds to c
    | RegExp :+: RegExp -- corresponds to e + f
    | RegExp :>: RegExp -- corresponds to e ; f
    | Star RegExp -- corresponds to e*
```

This allows us to describe regular expressions directly using the data types. For example, the regular expression

\[
a ; (b + c)^* ; a
\]

corresponds to

\[
\text{Chr 'a' :>: Star (Chr 'b' :+: Chr 'c') :>: Chr 'a'}
\]

Obviously your program will accept the second format.

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**Answer everything.**

1. Write a function `multiple`, which given a number \(n\) and a regular expression \(e\), returns a new regular expression that corresponds to regular expression \(e\) repeated \(n\) times in sequence. For example `multiple 3 (Star Any)` would return `Star Any :>: Star Any :>: Star Any`.

2. The minimum length of a string matching a regular expression can be calculated recursively. Clearly, ? and c always match a string of length 1. The regular expression `e+f` matches with a string which is at least as long as the smaller of the minimum lengths matched by e and f. The star operator `e*` can match the empty string, and thus has a minimum length of zero. Finally, with `e;f` the minimum length would be the sum
of the minimum lengths. Define a function \texttt{minLen} which given a regular expression calculates the minimum length of a matching string.

3. As already explained, parsing a string with respect to a regular expression involves writing a function which partially consumes a string, possibly finding no match:

\begin{verbatim}
parse :: RegExp -> String -> Maybe (String, String)
\end{verbatim}

Define \texttt{parse} using pattern matching on the regular expression. The operators \texttt{?} and \texttt{c} are easy to define. The case of \texttt{e;f} is slightly more complicated, since you should parse (recursively) the given string with respect to \texttt{e} and then parse the remaining unparsed string with respect to \texttt{f} (again, recursively). If the parsing of \texttt{e} fails, then the parse of \texttt{e;f} would completely fail, too. In the case of \texttt{e+f}, you will have to try both \texttt{e} and \texttt{f}, and then choose the one with the longest match (failing if both parses fail). Finally, the most complex operator is \texttt{e*}. The trick is to parse it as though you are parsing \texttt{e;e* + #}. The recursion would finally stop when the whole input is consumed\(^1\).

4. Now use the \texttt{parse} function to define a new function \texttt{match}, which given a regular expression and a string, returns whether the whole string matches the regular expression (ie leaving no part of the string unparsed).

If some things are unclear or you have some major problems, you can post your query on the discussion group at \url{http://groups.google.com/group/functional-programming-2007}. You need a gmail account to post your queries. Just in case you do not have one, e-mail Joseph Cordina at joseph.cordina(at)um.edu.mt or Claudia Borg at claudia.borg(at)um.edu.mt. Do not e-mail us directly with queries since we will not answer. Also do not post code or partial answers on the discussion board, since this will be penalized.

Good luck!

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\(^1\)There is one case in which this would not work, but for the assignment we do not really care! Can you identify it?