Expert Systems

(Rules of Inference)

Lecture 002

Sandro Spina

Propositional Logic

- Propositional logic offers another means of describing arguments. In fact, we often use propositional logic without realizing it !! For example:
  - If there is power, the computer will work
  - There is power
  - Therefore The computer will work
- In other words … if,
  - A = There is power
  - B = The computer will work
  - We can write down,
  - $A \rightarrow B$
  - A
  - B

Modus Ponens : “Way to assert”

- The argument of *modus ponens* $(p \rightarrow q, p; \quad q)$ is valid because it can be expressed as a tautology, i.e.
  - $(p \rightarrow q) \land p \rightarrow q$

p	q	$p \rightarrow q$	$(p \rightarrow q) \land p$	$(p \rightarrow q) \land p \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Modus Tollens : "way to deny"

- Law of contraposition (modus tollens)
  - $p \rightarrow q$
  - $\neg q$
  - $\quad \neg p$
- So if the conclusion is not true we can show that the premise was not true as well.

Laws of Inference (cont)

- Law of Detachment
  - $p \rightarrow q$
  - p
  - $\quad q$
- Law of Contrapositive
  - $p \rightarrow q$
  - $\quad \neg q \rightarrow \neg p$
- Chain Rule
  - $p \rightarrow q$
  - $q \rightarrow r$
  - $\quad p \rightarrow r$
- Law of Disjunctive Inference
  - $p \vee q$
  - $\neg p$
  - $\quad q$

Laws of Inference (cont)

- Law of Double Negation
  - $\neg(\neg p)$
  - p
- De Morgan's Law
  - $\neg(p \land q)$
  - $\neg p \lor \neg q$

  - $\neg(p \lor q)$
  - $\neg p \land \neg q$
- Law of Simplifications
- Law of Conjuction
- Law of Disjunctive Addition
- Law of Conjunctive Argument
  - $\neg(p \land q)$
  - p
  - $\neg q$

Limitations of Propositional Logic

- Consider the classic argument:
  - All men are mortal
  - Socrates is a man
  - Socrates is mortal

  - Using propositional logic we can get:
    - p = all men are mortal
    - q = Socrates is a man
    - r = Socrates is mortal

    - But the problem is that there are no logic connectives in the premise or conclusions and so each premise and each conclusion must have a different logical variable !

    - There are no quantifiers !!

Predicate Logic

All S is P	(Vx) (S(x) → P(x)
No S in P	(Vx) (S(x) → ¬P(x))
Some S is P	(x) (S(x) Λ P(x))
Some S is not P	(x) (S(x) Λ ¬P(x))

Hence consider the classic argument again :

- (Vx) (H(x) → M(x))
- H(s)
- H(s) → M(s) Universal Instantiation
- M(s) Modus Ponens

Hilbert Systems

- Each Hilbert system is defined by a collection of *axiom schemes* and a collection of *inference rules.* It is possible to set up a Hilbert system for propositional logic, which derives exactly all tautologies in propositional logic. For ex:

- Let a,b and c denote arbitrary propositional wff's
  - Axiom Schemes
    - $(a \rightarrow (b \rightarrow a))$; (note correction here !!!)
    - $((a \rightarrow (b \rightarrow c)) \rightarrow ((a \rightarrow b) \rightarrow (a \rightarrow c)))$
    - $(\neg b \rightarrow \neg a) \rightarrow ((\neg b \rightarrow a) \rightarrow b)$
  - Inference Rules
    - Modus Ponens

Hilbert Systems Example

- Consider the following hypotheses:
  - { A, A → B, B → C }
  - We need to show that (¬D → C)

- 1. A
- 2. A → B
- 3. B
- 4. B → C
- 5. C
- 6. C → ((¬D) → C)) (axiom schema)
- 7. ((¬D) → C)